

# Characterization of Multiconductor Coupled Lines from Multiport TDR Measurements

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**Abstract**— A new procedure to characterize nonuniform coupled multiconductor lines based on one dimensional peeling algorithm is presented. The measured multiport reflection (scattering parameter) parameters in time domain are used to construct piece-wise uniform configuration oriented models for multiple coupled lines. Examples of nonuniform two and three coupled striplines are included to illustrate the technique.

## INTRODUCTION

Measurement based characterization methods in time and frequency domain are now widely accepted techniques for accurate electrical characterization of high density interconnects [1–4]. The fast rise time of signals in digital systems and high frequencies in RF and microwave communication systems, lead to distributed circuit models of interconnects which are required for the simulation and reliable design of such systems.

Recently, an extraction procedure based on dynamic deconvolution [5] of discrete transmission-lines (layer peeling, Schur type difference equation) was developed to characterize general interconnects and discontinuities. The method has been extended to multiple-coupled transmission lines by formulating the algorithm for multidimensional systems[6]. The method, however, is computationally intensive and may become numerically unstable for more than two coupled transmission lines [3–7].

In this paper an efficient algorithm for the extraction procedure of required modal admittance profile from the measured data in time domain is presented for nonuniform multiconductor coupled transmission lines in a homogeneous medium. The inversion procedure utilizes one dimensional layer-peeling algorithm to multiconductor nonuniform coupled transmission lines TDR data and is based on the decomposition of admittance matrix by the suitable choice of eigenvectors. As an example, nonuniform coupled two and three striplines equivalent circuit, in terms of the cascaded sections of piecewise uniform transmission lines is extracted to demonstrate the procedure [8–10]. It is shown that the multiconductor nonuniform coupled lines can be characterized in terms of cascaded uniform coupled lines whose characteristic parameters are

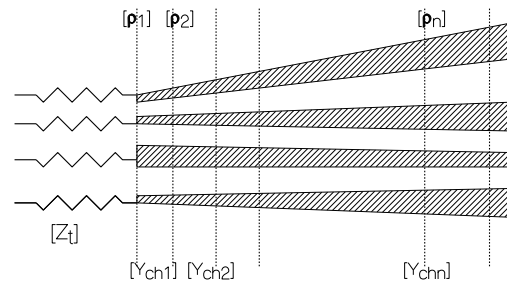


Fig. 1. The piecewise uniform model for nonuniform coupled lines.

extracted from the multiport time domain reflection measurements.

## COUPLED MULTICONDUCTOR NONUNIFORM LINES

The general model for nonuniform coupled lines consists of cascaded sections of piecewise uniform coupled lines (Fig. 1). Each section is characterized by a characteristic admittance (impedance) matrix whose element values can be extracted from the multiport TDR data. The expressions for the reflection coefficient matrices corresponding to each characteristic excitation (i.e., modal excitation) in terms of measured reflection coefficient matrix are used to extract these characteristic admittance values.

To characterize  $N$  nonuniform coupled transmission lines (with one dimensional peeling algorithm) we need to extract the characteristic admittance of each piecewise uniform line in the configuration oriented SPICE [8] model corresponding to that discrete incremental section. The measured voltage vector  $[v]$  at the interface of  $(n)$  and  $(n+1)$  section is given by,

$$[v] = [v_r] + [v_{in}] \quad (1)$$

and

$$[v_r] = [\rho][v_i], \quad (2)$$

where  $[v_r]$  represents the reflected and  $[v_{in}]$  represents the incidence voltage waveforms. At any interface, the admittance seen by the N port depends on the voltage vector. If the voltages corresponds to a modal value, the signal sees the corresponding modal admittance. For N TEM coupled lines excitation by 'even' mode (where all the voltages are equal), leads to the decoupling of N port and the characteristic admittance matrix reduces to a diagonal matrix, whose elements are the even mode admittance of individual lines. The measured data can be used to generate this desired characteristic excitation  $[v_{ch}]$ , at the interface of (n) and (n+1) section for which the required incidence voltage waveform  $[v_{ich}]$  is given as,

$$[v_{ich}] = [1 + [\rho]]^{-1} [v_{ch}]. \quad (3)$$

From equations (1), (2) and (3), the virtual modal reflection coefficient for the desired excitation  $[v_{ch}]$ , is found to be,

$$[v_{rch}] = [\rho_{virtual}][v_{ch}] = [\rho][1 + [\rho]]^{-1} [v_{ch}]. \quad (4)$$

For extracting transmission line admittance ( $Y_{ij}$ ), connecting port i and j the choice of excitation vector  $[v_{ch}]$  is,  $v_{ch}(i) = \alpha$  and  $v_{ch}(k \neq i) = 0, (k = 1, N)$ . The value of constant  $\alpha$ , is chosen so that  $v_{ich}(i) = 1$  in equation (3). Similarly for the extraction of self terms of transmission line admittance matrix, ( $Y_{ii} - \sum_{j=1, i \neq j}^N Y_{ij}$ ) connecting port i and 0 the choice of excitation vector  $[v_{ch}]$ , is given by,  $v_{ch}(i) = \alpha, (i = 1, N)$  and the constant  $\alpha$  is such that  $v_{ich}(i) = 1$  in equation (3).

The modal reflection coefficient at the sending end can be related to modal local reflection coefficient by one dimensional dynamic deconvolution relation [5] for each excitation eigenvector and thereby admittance associated with each port can be extracted in a sequential manner.

The algorithm can be summarized as

- Measure the elements of  $[\rho]$  matrix.
- Characterize section one by finding all the elements of  $[y_{ch}]_1$  matrix from constructing  $[\rho]_m, m = 1, \dots, N$ .
- Use  $[\rho]_m$  data corresponding to even mode (all port voltages are equal and the characteristic admittance

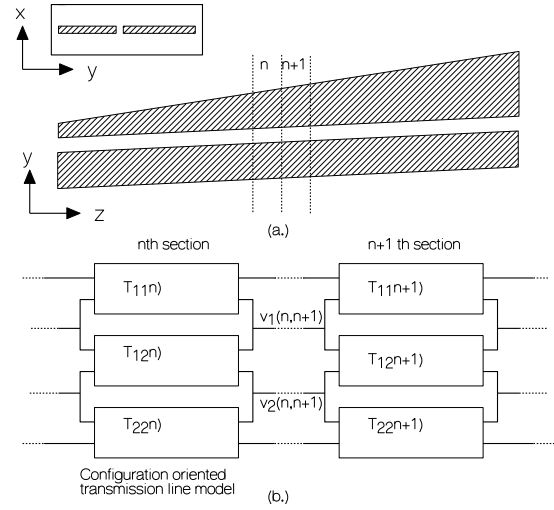


Fig. 2. a.) Nonuniform coupled interconnect in homogeneous medium. b.) Equivalent electrical model using cascaded uniform configuration oriented coupled line model.

matrix of the extracted section is diagonal) for subsequent deconvolving the section.

- Repeat the procedure.

It is noted that in general N nonuniform coupled lines require  $N(N+1)/2$  one dimensional dynamic deconvolution procedures for the complete characterization of coupled lines system.

#### A. Coupled Asymmetric Lines: Homogeneous Medium

For asymmetric coupled lines in a homogeneous medium (Fig 2.a,b), the choice of even and odd mode excitation, ( $v_{1,n,n+1} = v_{2,n,n+1}$  and  $v_{1,n,n+1} = -v_{2,n,n+1}$ ), leads to the modal reflection coefficients for even and odd mode in terms of local reflection coefficient matrix ( $\rho_{11,n,n+1}, \rho_{12,n,n+1}, \rho_{22,n,n+1}$ ) at nth and n+1th interface and are given by,

$$\begin{aligned} \rho_{1e,n,n+1} &= \rho_{11,n,n+1} + \rho_{12,n,n+1} \frac{1 + \rho_{11,n,n+1} - \rho_{21,n,n+1}}{1 + \rho_{22,n,n+1} - \rho_{12,n,n+1}} \\ \rho_{2e,n,n+1} &= \rho_{22,n,n+1} + \rho_{21,n,n+1} \frac{1 + \rho_{22,n,n+1} - \rho_{12,n,n+1}}{1 + \rho_{11,n,n+1} - \rho_{21,n,n+1}} \\ \rho_{1o,n,n+1} &= \rho_{11,n,n+1} - \rho_{12,n,n+1} \frac{1 + \rho_{11,n,n+1} + \rho_{21,n,n+1}}{1 + \rho_{22,n,n+1} + \rho_{12,n,n+1}} \\ \rho_{2o,n,n+1} &= \rho_{22,n,n+1} - \rho_{21,n,n+1} \frac{1 + \rho_{22,n,n+1} + \rho_{12,n,n+1}}{1 + \rho_{11,n,n+1} + \rho_{21,n,n+1}} \end{aligned} \quad (5)$$

The admittance are related to the reflection coefficients by,

$$y_{e/o,1/2}(n+1) = \frac{1 - \rho_{1/2,e/o,n,n+1}}{1 + \rho_{1/2,e/o,n,n+1}} y_{e/o,1/2}(n). \quad (6)$$

The reflection coefficient at sending end can be related to local reflection coefficient and in general four one dimensional dynamic deconvolution procedure are used recursively to extract time dependent nonuniform profile of characteristic admittance matrix elements. For each section

$$y_{ch}(n+1) = \begin{bmatrix} \frac{y_{e1}(n+1) + y_{o1}(n+1)}{2} & \frac{y_{e1}(n+1) - y_{o1}(n+1)}{2} \\ \frac{y_{e2}(n+1) - y_{o2}(n+1)}{2} & \frac{y_{e2}(n+1) + y_{o2}(n+1)}{2} \end{bmatrix}. \quad (7)$$

The extracted characteristic admittance matrix is symmetric therefore the four one dimensional dynamic deconvolution procedure used above can be reduced to three.

### B. Coupled Three Lines: Homogeneous Medium

In case of nonuniform coupled three lines the choice of desired characteristic excitation vector  $[v_{ch}]$  are four vectors given by (1,1,1), (1,0,0), (0,1,0), (0,0,1). The value of constant  $\alpha$  is chosen for each excitation from equation (3) and the virtual modal reflection coefficient for the desired excitation are chosen by equation (4). Six one dimensional deconvolution procedure are required for complete extraction of equivalent circuit for coupled three lines system.

## RESULTS

In order to demonstrate the usefulness of the extraction procedure for typical two and three nonuniform coupled striplines structures are presented. For the four port shown in Fig. 3a the measured 2 port TDR voltage response of the nonuniform coupled transmission line interconnect is shown in Fig. 3b. Fig. 3c shows the extracted impedance profile by one dimensional peeling algorithm presented here and by the two dimensional peeling algorithm[6]. The admittance profiles, which completely characterize the coupled system, extracted by the both techniques are virtually identical validating the algorithm developed in this paper. For a three coupled line case, SPICE simulated TDR response is used to exemplify the procedure. Fig. 4b shows the simulated TDR voltage response for symmetric three coupled lines as shown in Fig. 4a. Fig. 4c shows the extracted impedance profile by one dimensional peeling algorithm presented here and is same as that of the simulated structure shown in Fig. 4a. The computational time taken by this algorithm is an order of magnitude smaller than two or three dimensional peeling algorithm. In addition, it should be noted that the multidimensional peeling algorithm becomes quite tedious

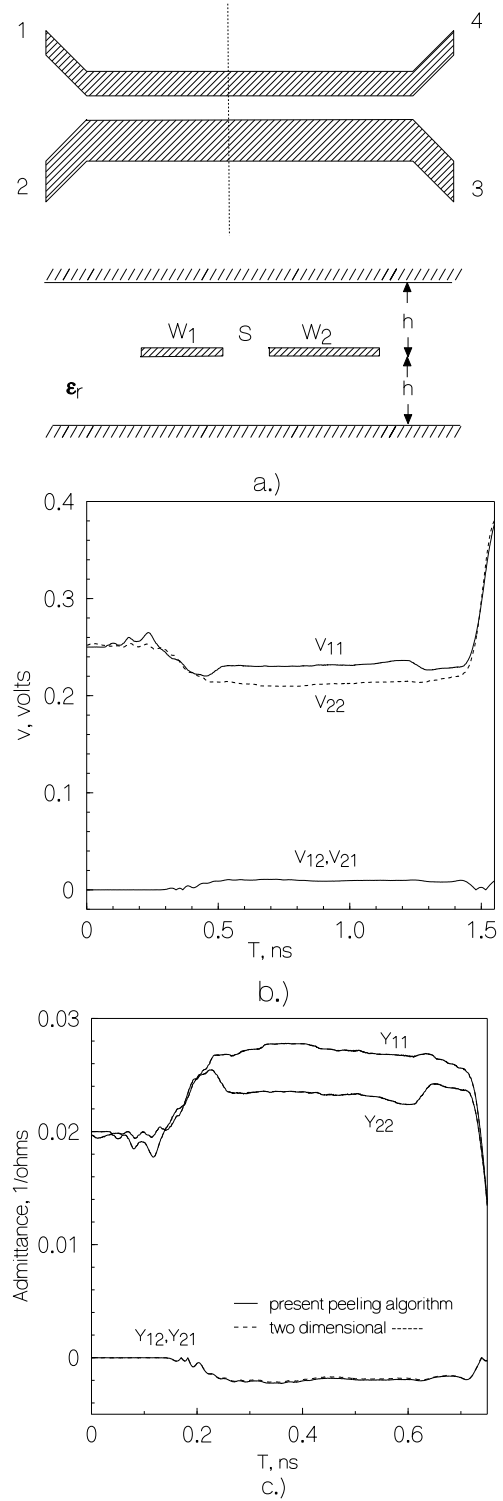


Fig. 3. a) Nonuniform coupled interconnect, b) Measured voltage response due to step excitation for nonuniform coupled interconnect, c) Extracted admittance profile 'y11,y22,Y12' with two dimensional peeling algorithm and by the procedure in paper. (w1=3.5mm, w2=4.5mm, s=4.0mm, h=2.2mm,  $\epsilon_r = 4$ )

and unmanageable for more than 3 coupled lines making the technique presented here compatible with the systems having arbitrary large number of lines.

## CONCLUSION

The measurement based technique to characterize uniform and nonuniform multiconductor lines have been presented. The efficient techniques are based on one dimensional deconvolution algorithms and are suitable for multiconductor interconnect and transmission lines structures in high speed digital and RF/microwave circuits and system.

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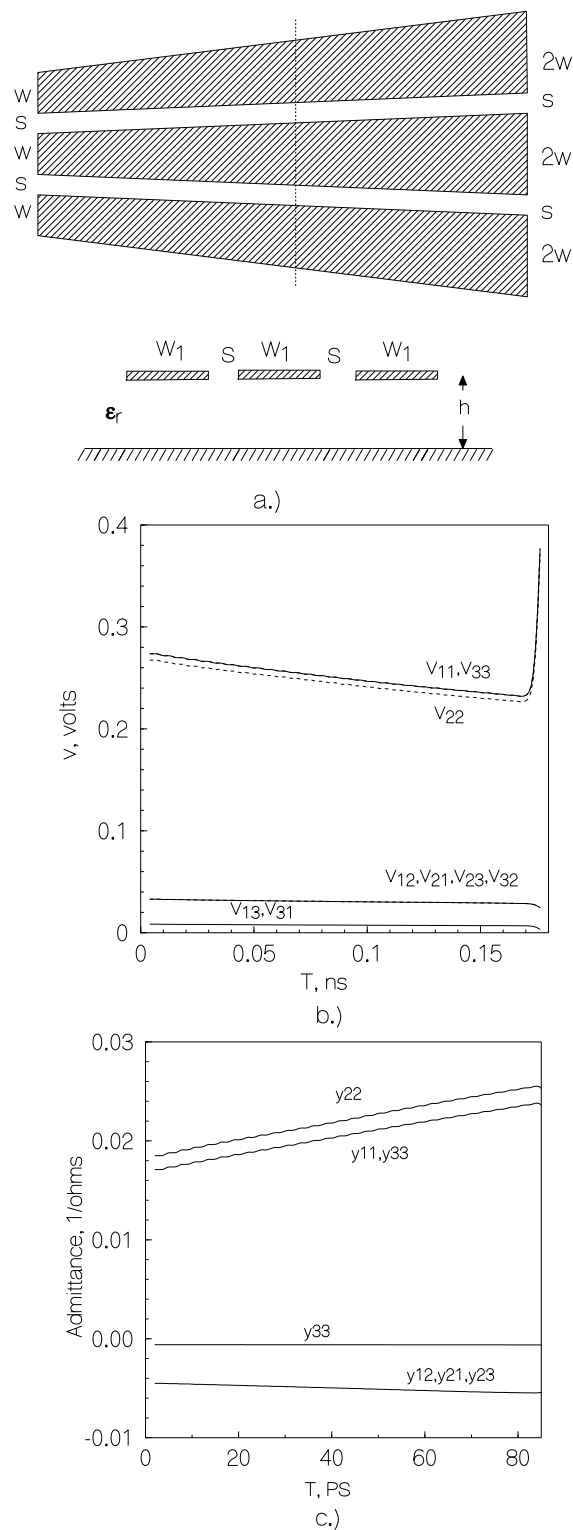


Fig. 4. a.) Nonuniform symmetric three line coupled interconnect, b) Simulated voltage response due to the step excitations, c) Extracted Admittance profile by the procedure described in paper. ( $w=1\text{mm}$ ,  $s=0.5\text{mm}$ ,  $\epsilon_r = 4$ ,  $h=1\text{mm}$ )